International Journal of Research in Advent Technology, Vol.6, No.5, May 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Axially Symmetric Cosmological Models In $f(R,T)$ Gravity With Time Varying Deceleration Parameter

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Abstract: In the present paper an attempt has been made to study an axially symmetric space-time is considered in the presence of a perfect fluid source in the framework of $f(R,T)$ gravity. We consider two types of scale factors (i) $a(t)=\sinh(\alpha t)$ and (ii) $a(t)=te^{t}$ which yield time dependent DP. To get deterministic solution, expansion scalar θ is proportional to the shear scalar σ have been used. Some physical behavior for both models have been discussed by using physical parameters.

Keywords: $f(R, T)$ gravity, Axially symmetric space-time, Deceleration Parameter

1. INTRODUCTION

During last decade, there has been several modifications of general relativity to provide natural gravitational alternative for dark energy. It has been suggested that cosmic acceleration can be achieved by replacing the Einstein-Hilbert action of general relativity with a general function Ricci scalar, *f (R)*. Modification of general relativity are attracting more and more attention to explain late time acceleration and dark energy. There have been several modified theories like *f* (*R*) gravity, *f* (*G*) gravity f(T) gravity or *f* (*R*,*G*) gravity and so on investigated by several researchers. A generalization of *f* (*R*) modified theories of gravity was proposed in [1]. Among the various modifications, *f* (*R*) theory of gravity is considered as the most suitable due to the cosmologically important *f* (*R*) models. It has been suggested that cosmic acceleration can be achieved by replacing the Einstein–Hilbert action of general relativity with a general function Ricci scalar, *f* (*R*). Nojiri *et al* [2] have studied *f* (*R*), *f* (*G*) or *f* (*R*,*G*) gravity in various contexts. Many researchers [3–11] have investigated *f* (*R*) gravity in different contexts. Shamir [12] has proposed a physically viable *f* (*R*) gravity model, which shows the unification of early-time inflation and late-time acceleration. Paul et al. [13] obtained FRW models in f(R) gravity while Sharif and Shamir [14,15] have studied the solutions of Bianchi type I and V space-times in the frame work of f(R) gravity. Shamir [16] studied the exact vacuum solutions of Bianchi type I, III and Kantowski-Sachs space-times in the metric version of f(R) gravity. Adhav [17] has obtained LRS Bianchi type I cosmological model in f(R, T) gravity. Reddy et al. [18] have discussed Bianchi type III cosmological model in f(R, T) while Reddy et al. [19], Reddy and Shanthikumar [24] studied Bianchi type III dark energy model and some anisotropic cosmological models, respectively, in f(R, T) gravity. Chaubey and Shukla [20] have constructed a new class of Bianchi cosmological model in f(R, T) gravity. The exact solutions of the Einstein-Rosen cosmological model filled with perfect fluid have been derived in f(R, T) gravity by Rao and Neelima [21].

Axially symmetric cosmological models with string dust cloud source developed by Bhattacharaya and Karade [22]. Axially symmetric space-times representing material distribution were obtained by Marder [23].

Axially symmetric space-times play an important role in the study of universe on a scale in which anisotropy and inhomogeneity are not ignored [24]. Kilinc [25] showed that axially symmetric cosmological models have made significant contributions in understanding some essential features of the universe such as the formation of galaxies during the early stages of the evolution. Jain et al. [26] studied axially symmetric space-time with wet dark fluid in bimetric theory. Energymomentum localization in Marder's axially symmetric space-time has been studied by Aygun et al. [27]. Recently Rao et al. [28] have obtained axially symmetric cosmological model with perfect fluid in general relativity and in f(R, T) gravity. Sahoo et al. [29] obtained the exact solution of the field equations in f(R, T) theory with the help of special law of variation for Hubble parameter. Two fluid axially symmetric cosmological models in f(R, T) theory obtained by Pawar et al. [30].

The constant DP is commonly used by cosmologist in literature with various aspects. In order to make more detailed description of the kinematics of cosmological expansion, it is useful to consider various forms of time dependence deceleration parameter. One of the most popular form is known as linearly varying deceleration parameter (LVDP). Linear parametrization of the DP represents quite naturally. The next logical step towards the behaviour of future model is either it expands forever or ends with a Big Rip in finite future. This can be parametrized with redshift parameter z, cosmic scale factor a and with cosmic time t. Transition of the universe from the decelerated phase to the present accelerating phase motivate us to consider variable DP. Several researchers (Pradhan *et al.* [31], Amirhashchi *et al.* [32], Akarsu and Dereli [33]) have discussed the evolution of the universe with variable DP. Also Mishra et al [34] have proposed a linearly varying deceleration parameter and using it they have investigated Bianchi Type-II dark energy model in f(R, T) gravity. It is used in Bianchi type-V cosmological model with of holographic dark energy to escape the Big Rip singularity [35]. Singh et al.[36] have been studied the homogeneous and anisotropic Bianchi type-I cosmological model in the presence of viscous uid source of matter, which starts with a big bang and ends in a Big Rip. The kinematical behaviour of LVDP along with null energy condition (NEC) has been explored in the framework of f(R; T) gravity for Bianchi type -I and V space-time[37]. Akarsu et al.[38]have described the fate of the universe through parametrization q = $q0+q1(1-t/t_0)$, which is linear in cosmic time t, along with two well-known additional parametrization of the DP $q = q_0+q1(1-a/a_0)$ and $q = q_0 + q_1 z$, where z and a are the redshift parameter and scale factor respectively. Furthermore, they have studied the dynamics of the universe in comparison with the standard ^CDM model. Sahoo and Sivakumar [39] have obtained the model for perfect fluid source coupled with strange quark matter with linearly cosmic time parametrization of the deceleration parameter.

The present paper is organized as follows. In sect.1, a brief introduction is given. The field equations in metric version of $f(R, T)$ gravity is given in sect. 2. In section 3, explicit field equations in $f(R, T)$ gravity are obtained by using the particular form of the functions $f(T) = \lambda T$, which are used by Harko et al. [40], with the general class of axially symmetric metric in the presence of perfect fluid. Section 4 deals with cosmological solutions of the field equations using the linearly varying deceleration parameter by considering physically relevant assumptions and also discuss some physical properties of the model. Conclusions are given in sect.5.

2. GRAVITATIONAL FIELD EQUATIONS OF $f(R,T)$ GRAVITY

The $f(R,T)$ theory of gravity is the modifications of General Relativity (GR). The field equations of $f(R,T)$ gravity are derived from the Hilbert-Einstein type variational principle. The action for the modified $f(R,T)$ gravity is

$$S=\frac{1}{16\pi}\int\left[f(R,T)+L_m\right]\sqrt{-g}\,d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, T is the stress energy tensor T_{ij} of the matter and L_m is the matter Lagrangian density, The energy momentum tensor T_{ij} is defined as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\right)L_m}{\delta g^{ij}}. \tag{2}$$

Here we assume that the dependence of matter Lagrangian is merely on the metric tensor g_{ij} rather than its derivatives.

In this case, we obtain

$$T_{ij}=g_{ij}L_m-\frac{\partial L_m}{\partial g^{ij}}. \tag{3}$$

The $f(R,T)$ gravity field equations are obtained by varying the action S with respect to metric tensor g_{ij}.

$$f(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+\left(g_{ij}\otimes-\nabla_i\nabla_j\right)f_R(R,T)=8\pi T_{ij}-f_T(R,T)T_{ij}- \tag{4}$$

where $\theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{\alpha\beta}\dfrac{\partial L_m}{\partial g^{ij}\partial g^{\alpha\beta}}$. (5)

Here

$$f_R(R,T)=\frac{\partial f(R,T)}{\partial R},\quad f_T(R,T)=\frac{\partial f(R,T)}{\partial T},\ \otimes=\nabla^{i}\nabla_{i}$$

where ∇_i denotes covariant derivative.

Now contraction of equation (4) gives

$$f_R(R,T)R+3\otimes f_R(R,T)g_{ij}-2f(R,T)=8\pi T-f_T(R,T)(T+\theta), \tag{6}$$

Where $\theta=\theta_i^i$. Equation (6) gives a relation between Ricci scalar R and the trace T of energy momentum tensor. Using matter Lagrangian L_m the stress energy tensor of the matter is given by

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij}, \tag{7}$$

where $u^i=(0,0,0,1)$ is the four velocity in commoving coordinates which satisfies the condition $u^iu_i=1$ and $u^i\nabla_ju_i=0$. ρ and p are energy density and pressure of the fluid, respectively, and the matter Lagrangian can be taken as $L_m=-p$ since there is no unique definition of the matter Lagrangian. Then with the use of (5) we obtain the variation of stress energy of perfect fluid expression

$$\theta_{ij} = -2T_{ij} - pg_{ij}, \tag{8}$$

On the physical matter of the matter field, the field equations also depend through the tensor θ_{ij}. Hence in the case of $f(R,T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to different matter contribution for $f(R,T)$ gravity are possible. However, Harko et al. [40] gave three cases of these models.

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} \tag{9}$$

In this paper, we are focused to the first class, i.e. $f(R,T) = R + 2f(T)$ where $f(T)$ is an arbitrary function of stress energy tensor of matter. We get the gravitational field equations of $f(R,T)$ gravity from equation (4) as

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T) T_{ij} - 2f'(T)\theta_{ij} + 2f(T) g_{ij} \tag{10}$$

where the prime denotes differentiation with respect to the argument. If the matter source is a perfect fluid then the field equations (in view of equation (8)) becomes

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f'(T) T_{ij} + \left[2pf'(T) + f(T)\right] g_{ij} \tag{11}$$

3. METRIC AND FIELD EQUATIONS

We consider the axially symmetric metric (Bhattacharya and Karade [22]) as

$$ds^2 = dt^2 - A^2(t)\left[d\chi^2 + f^2(\chi) d\phi^2\right] - B^2(t) dz^2, \tag{12}$$

with the convention $x^1 = \chi,\ x^2 = \phi,\ x^3 = z \ \ and \ \ x^4 = t,\ and \ \ A, B$ are functions of the proper time t alone while f is a function of the coordinate χ alone . In view of equation (7) for axially symmetric space-time (12) , the field equations (11) leads to

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = p(8\pi + 3\lambda) - \rho\lambda, \tag{13}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{f''}{A^2 f} = p(8\pi + 3\lambda) - \rho\lambda, \tag{14}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} - \frac{f''}{A^2 f} = -\rho(8\pi + 3\lambda) + p\lambda, \tag{15}$$

where an overhead dot and dash represents differentiation with respect to t and χ respectively.

The functional dependence of the metric together with equation (14) and (15) imply that

$$\frac{f''}{f} = m^2, \quad m^2 = constant. \tag{16}$$

If $m = 0$, then $f(\chi) = c_1\chi + c_2, \quad \chi > 0$ (17)

where c_1 *and* c_2 are integrating constants. Without loss of generality, by taking $c_1 = 1$ *and* $c_2 = 0$ we get $f(\chi) = \chi$ resulting in the flat model of the universe (Hawking and Ellis [41]).

Now the field equations (13)-(15) reduces to

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = p(8\pi + 3\lambda) - \rho\lambda, \tag{18}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 = p(8\pi + 3\lambda) - \rho\lambda, \tag{19}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = -\rho(8\pi + 3\lambda) + p\lambda. \tag{20}$$

The spatial volume is given by

$$V = a^3 = A^2B \tag{21}$$

where a is the mean scale factor.

The mean Hubble parameter H for axially symmetric metric is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left[2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right] \tag{22}$$

The directional Hubble parameter in the directions of χ, ϕ *and* z are

$$H_\chi = H_\phi = \frac{\dot{A}}{A} \text{ and } H_z = \frac{\dot{B}}{B} \tag{23}$$

The deceleration parameter is

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{24}$$

4. SOLUTIONS OF THE MODEL

Equations (18)-(20) are three independent equations in four unknowns A, B, p, ρ . Hence to find a determinate solution, we assume expansion scalar θ is proportional to the shear scalar σ which yields

$$A = B^n \tag{25}$$

Subtracting (18) from (19) and taking second integral, we obtain the following relation

$$\frac{A}{B} = k_2 e^{k_1\int \frac{1}{a^3}dt} \tag{26}$$

Thus equation (26) gives values of A, B as

$$A = k_2^{\frac{n}{(n-1)}} e^{\frac{k_1 n}{(n-1)}\int \frac{1}{a^3} dt} \quad n \neq 1$$

(27)

$$B = k_2^{\frac{1}{(n-1)}} e^{\frac{k_1}{(n-1)}\int \frac{1}{a^3} dt} \quad n \neq 1$$

(28)

Thus the metric functions are found explicitly in terms of average scale factor a.

4.1. *Case I:* $a(t) = \sinh(\alpha t)$

As suggested by Pradhan et al. [42], firstly we consider the variation of scale factor a with cosmic time t by the relation

$$a(t) = \sinh(\alpha t)$$

(29)

where α is an arbitrary constant.

Using equation (29) into (27) and (28), we have the following set of expressions for the scale factors

$$A = k_2^{\frac{n}{(n-1)}} e^{\frac{k_1 n}{(n-1)}\int [\sinh(\alpha t)]^{-3} dt} \qquad n \neq 1$$

(30)

$$B = k_2^{\frac{1}{(n-1)}} e^{\frac{k_1}{(n-1)}\int [\sinh(\alpha t)]^{-3} dt} \qquad n \neq 1$$

(31)

The physical quantities of observational interest in cosmology such as directional Hubble parameters (H_i), spatial volume (V), mean anisotropy parameter (Δ), shear scalar (σ^2) and expansion scalar (θ) are respectively given by

$$H_\chi = H_\phi = \frac{k_1 n}{(n-1)\sinh(\alpha t)}$$

(32)

$$H_z = \frac{k_1}{(n-1)\sinh(\alpha t)}$$

(33)

$$V = \sinh^3(\alpha t)$$

(34)

$$\Delta = \frac{2}{3}\frac{(3n^2 + 2n + 1)}{(2n+1)^2}$$

(35)

$$\sigma^2 = \frac{k_1^2 (3n^2 + 2n + 1)}{(n-1)^2 \sinh^2(\alpha t)}$$

(36)

$$\theta = \frac{3k_1(2n+1)}{(n-1)\sinh(\alpha t)}$$

(37)

The deceleration parameter as

$$q = -\tanh^2(\alpha t)$$

(38)

Using equations (30) and (31) in (19) and (20), we obtain the values of pressure and energy density as

$$p = \frac{k_1 n}{(n-1)^2 \sinh^2(\alpha t)\left[(8\pi + 3\lambda)^2 - \lambda^2\right]}\left[(8\pi + 3\lambda)(3k_1 n - 2\alpha(n -\right.$$

(39)

$$\rho = \frac{k_1 n}{(n-1)^2 \sinh^2(\alpha t)\left[(8\pi+3\lambda)^2-\lambda^2\right]}\left[\lambda(3k_1 n-2\alpha(n-1)\cosh(\alpha t))-(8\pi+3\lambda)k_1(2+n)\right]$$

(40)

4.2. Physical Behavior of the model

When $t=0$ the scalar expansion and shear scalar are infinity but at $t=\infty$ the scalar expansion and shear scalar are zero. The model of the universe start with big bang Spatial volume expands exponentially as t increases and becomes infinitely large as $t\to\infty$.The directional Hubble parameters $H_\chi,\ H_\phi,\ H_z$ are infinite at $t=0$ and vanishing at $t\to\infty$.The shear scalar become zero as $t\to\infty$. We observe pressure and energy density remains always positive and it converges to zero as $t\to\infty$. From equations (30) and (31) we observe that the spatial scale factor become constant at the initial epoch $t=0$. Since $\lim_{t\to\infty}\frac{\sigma}{\theta}\neq 0$ the model does not approach isotropy for large value of t.

4.3. CASE II: $a(t)=te^t$

Following Amirhashchi et al.[43], we consider the following scale factor

$$a(t)=te^t$$

(41)

Using equations (41) in (27) and (28), we have the following set of expressions for the scale factors

$$A = k_2^{\frac{n}{(n-1)}} e^{\frac{k_1 n}{(n-1)}\int\left[te^t\right]^{-3}dt} \quad n\neq 1$$

(42)

$$B = k_2^{\frac{1}{(n-1)}} e^{\frac{k_1}{(n-1)}\int\left[te^t\right]^{-3}dt} \quad n\neq 1$$

(43)

The physical parameters as described in the case I are expressed as

$$H_\chi = H_\phi = \frac{k_1 n}{(n-1)t^3 e^{3t}}$$

(44)

$$H_z = \frac{k_1}{(n-1)t^3 e^{3t}}$$

(45)

$$V = t^3 e^{3t}$$

(46)

$$\Delta = \frac{2}{3}\frac{(3n^2+2n+1)}{(2n+1)^2}$$

(47)

$$\sigma^2 = \frac{k_1^2(3n^2+2n+1)}{(n-1)^2 t^6 e^{6t}}$$

(48)

$$\theta = \frac{3k_1(2n+1)}{(n-1)t^3 e^{3t}}$$

(49)

The deceleration parameter as

$$q = -\frac{t(t+2)}{(t+1)^2}$$

(50)

Using equations (42) and (43) in (19) and (20), we obtain the values of pressure and energy density as

$$p = \frac{k_1 n}{(n-1)^2 t^6 e^{6t}\left[(8\pi+3\lambda)^2-\lambda^2\right]}\left[(8\pi+3\lambda)\left(-6(n-1)(t+1)t^2 e^{3t}+3k_1 n\right)-\lambda k_1(2+n)\right] \quad (51)$$

$$\rho = \frac{k_1 n}{(n-1)^2 t^6 e^{6t}\left[(8\pi+3\lambda)^2-\lambda^2\right]}\left[\lambda\left(-6(n-1)(t+1)t^2 e^{3t}+3k_1 n\right)-(8\pi+3\lambda)k_1(2+n)\right] \quad (52)$$

4.4. *Physical Behavior of the model*

We observe that the spatial volume is zero at $t = 0$ and expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario. We observe that the spatial volume increases exponentially with time. The physical quantities pressure (p), energy density (ρ), Hubble factor (H), shear scalar (σ^2) and expansion scalar (θ) diverge at $t = 0$. As $t \to \infty$ volume becomes infinite whereas p, ρ, H, θ approaches to zero. We observe that the average scale factor $a(t) \to 0$ as $t \to 0$ and $a(t) \to \infty$ as $t \to \infty$ .This indicates that there exists inflation. Since $\lim_{t\to\infty} \frac{\sigma}{\theta} \neq 0$ the model does not approach isotropy throughout the evolution of the universe.

5. CONCLUSION

In this paper, we have considered a cosmological model in the presence of perfect fluid and variable deceleration parameter in *f* (*R*, *T*) theory of gravity. According the choice of *f* (*R*, *T*), we are focused to the first class, i.e. $f(R, T) = R + 2f(T)$.

The exact solutions of the modified Einstein's field equations are obtained for the axially symmetric universe with perfect fluid. We assumed two types of scale factors (i) $a(t) = \sinh(\alpha t)$ and (ii) $a(t) = te^t$ which yield time dependent DP.

The observations of both the model are as follows:

The mean anisotropy parameter becomes constant for both models. It can be observed that our models are expanding and accelerating universe which starts at a big bang singularity. In both the cases energy density is positive valued and decreasing function of time. It is interesting to notice that q decreases very rapidly and then after it remains constant. It is observed that, our derived model has accelerated expansion at present epoch which is consistent with recent observation of type Ia supernova and CMB anisotropies.

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